RESEARCH

# A proof of Collatz's Conjecture

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## ABSTRACT

The proof of Collatz Conjecture is divided in to three parts. In the First part it is proved for the set of numbers {1, 5, 21, 85}.

In the second part, using the above it is proved for odd numbers of the form (4k+1). In the third part it is shown how numbers of the form (4k+3) iterates in to numbers of the form (4k+1). Thus the Conjecture is proved for all odd 'n'. As even numbers iterate into Odd numbers, the Conjecture is proved for all positive integers.


**Key Words:** *Collatz's Conjecture; Part; Positive; Number; Integers*

## INTRODUCTION

Consider the following equation for a positive integer 'n'.

$f(n,1) = n / 2$, if 'n' is even and $= (3n + 1)$ if 'n' is odd.

Let $f(n, r) = f[f(n, r - 1), 1]$ for $n \geq 2$

Now consider the set

$A(n) = \{ f(n,1) \text{-------}, f(n, r), \text{------} \}$

Consider an imaginary set

$B(n) = \{ f(n,1), \text{----}, 1, 4, 2, 1, 4, 2, 1, \text{--} \}$

According to Collatz's Conjecture, both the sets are identical.

That is A(n)=B(n)

Therefore, A(n)=B(n), means Collatz's Conjecture is true for 'n'. It is obvious that Collatz's Conjecture has to be proved for odd value of 'n', as for even value, it will iterate in to an odd number and ultimately ending in the odd number '1'.

**Par 1: An analysis of numbers 1, 5, 21, 85 etc.**

Consider the number $n = [\, 4^{r-1} + 4^{r-2} + 4^{r-3} + \text{———————} + 1]$

$f(n,1) = (4 - 1)[\, 4^{r-1} + \text{-------} + 1\,] + 1 = 4^r$

Therefore, f (n, 2r+1) = 1.

Hence A(n)=B(n).

Thus, a set $S_1 = \{1, 5, 21, 85, 341, \text{————————}\}$, is generated.

$S_1 = \{s_r : s_r = (4^r - 1)/3, \geq 1\}$

This set is the 'PRIMARY SET', because every other number iterates into a member of this set greater than '1' before iterating into '1'.

**Part 2: An analysis of n = 4k+1**

As only odd value of 'n' is analysed it can be divided into two forms, that is, n=4k+1 and n = 4k+3. An analysis of n=4k+3 is dealt in part 3. But, part 2 and part 3 are interconnected.

Consider, n=4k+1

f (n, 1) = f [(4k+1), 1]
= 12k+6

f (n, 2) = 6k+3

f (n, 3) = 3k+1

If 'k' is odd, then f (k) = 3k+1.

Or if 'n' is an odd number and A(n) = B(n),

Then, A(4n+1) = B(4n+1).

This is an important result and it can even derive the primary set.

$S_1 = \{1, 4+1, 5(4)+1, 21(4)+1, \text{——————}\}$

$S_1 = \{1, 5, 21, \text{————————————}\}$, similarly

$S_3 = \{3, 13, 53, \text{—————}\}$

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$S_7$={7, 29, 117, – – – – –}
$S_9$={9, 37, 149, – – – – –} Or in general
$S_{2n+1}$={$2n+1, 8n+5, 32n+21$, – – –}.

These sets are important in the understanding of the problem.
In these sets, except the first member, all the other members Are, obviously, of the (4k+1) form.
Consider the numbers below 21.
The proof for (3, 11, 19,--); (7, 23,--); (15,47-) etc are given in part-3. What is left out is '9 and 17' which are of the (8k+1) form, which iterate into (6k+1), which is of (4k+1) or (4k+3) form. Example is '9' and it is an interesting number, as it iterates into 7, 11, 17, 13, 5 and to '1'.
If 'n' is odd and A(n) = B(n), then A(4n+1) = B(4n+1).

The converse is also true.

If 'n' is odd and A(4n+1) = B(4n+1), then A(n) B(n).

Now, assume that, for $n \le (4k+1)$, $A(n) = B(n)$

Consider $n_1 = 4(k+r) + 1$.

If $(k+r)$ *is odd* & $(k+r) \le (4k+1)$, *then*
$(k+r) = (k+r)$ & *so* $(n_1) = (n_1)$.

If $(k+r)$ *is even* & $3(k+r)+1 \le (4k+1)$, *then* $A(n_1) = B(n_1)$.
That is, if, $3r \le k$, $(n_1) = (n_1)$.

**'k' is odd, 'r' is even & so 'k + r' is odd.**

$k = 1$, therefore $(k+r) \le 5$

Hence, $(k+r)$ = 3,5 and *so*, $A(3) = B(3)$, $A(13) = B(13)$.
(5) = (5) and $A(21) = B(21)$.

$k$=3, therefore $(k+r) \le 13$.

Hence, $(k+r)$ = 5, 7, 9, 11, 13.

$k = 5$, therefore $(k+r) \le 21$.

Hence, $(k+r)$ =7, 9, 11, 13, 15, 17, 19, 21 and so on.

**'k' is even, 'r' is odd and so 'k + r' is odd.**

$k$=2, therefore $(k+r) \le 9$.

Hence, $(k+r)$ =3, 5, 7, 9.

$k$=4, therefore $(k+r) \le 17$.

Hence, $(k+r)$ =9, 11, 13, 15, 17.

$k = 6$, therefore $(k+r) \le 25$.

Hence, $(k+r)$ =17, 19, 21, 23, 25,.
$k = 8$, therefore $(k+r) \le 33$.

Hence, $(k+r)$ =25, 27, 29, 31, 33 and so on.

These generates the set, $K_1$={1, 3, 5, 7, – – – – – – –}

$K_1$={ $k$: $k=2s+1, s \in N$}

**'k' is odd, 'r' is odd and so 'k + r' is even**

$(n_1) = (n_1)$, *if* $3r \ge$ k.

Hence, for, k=3, 5, 7; $r$ =1 and $(k+r)$=4, 6 , 8.

*for*, =9, 11, 13; $r$ =1,3 and $(k+r)$ =10, 12, 14, 16. And so on.

These generates the set

$k_2 = \{2s \,\&\, s \in N\}$

($k$ =0, 2 is include, as it can be verified ).

The set, $K = K_1 \cup K_2 = \{K : K \in N\}$.
This set' K' generates the set,
$N(2^2) = \{1, 5, 9, 13, \text{- - - -}\}$
$= \{n_2 : n_2 = (4k+1), k \in N\}$
$A(n_2) = B(n_2)$.

It may be noted that in the set 'K1', A(K) = B(k). Therefore, the set 'K1' itself proves Collatz's Conjecture. But, however part 3 is important, as the necessary connectivity is provided by it.

**Part 3: An analysis of n = (4k+3)**

Though it is assumed, n = 4k+1, is solved, 4k+1 and 4k+3 are interconnected. It will be shown how (4k+3) becomes (4k+1) Before becoming '1'.
Let, n = ( 4k + 3 ); 'k' can be odd and even.
Let, $k = 2k_3$ or $2k_3 + 1$
Therefore, $n_3 = (8k_3 + 3)$ or $n_{31} = (8k_3 + 7)$.
$n_3 = (8k_3 + 3)$
So, $f(n_3, 1) = f(2^3 k_3 + 2^2 - 1, 1) = 3.2^3 k_3 + 2^3 + 2^2 - 3 + 1$
$f(n_3, 2) = 3.2^2 k_3 + 2^2 + 2 - 1 = 4(3k_3 + 1) 4(3k_3 + 1) + 1$
Therefore, $A(n_3) = B(n_3)$
Thus, the following set is generated. Hence
$N(2^3) = \{3, 11, 19, \text{- - - -}\} = \{n_3 : n_{3'} = (2^3 k + 3), k^3 0\}$.
$n_{31}$, can be odd or even
Therefore let, $n_4 = 2^4 k_4 + 7$ & $n_{41} = 2^4 k_4 + 15$
$n_4 = 2^4 k_4 + 2^3 - 1$
$f(n_4, 2) = 3.2^4 k_4 + 2^4 + 2^3 - 3 + 1 = 2^4(3k_4 + 1) + 2^3 - 2$
$f(n_4, 2) = 2^3(3k_4 + 1) + 3$
$f(n_4, 4) = 2^2(3^2 k_4 + 3 + 1) + 1$.

Therefore, $A(n_4) = B(n_4)$

Thus the following set is generated

$N(2^4) = \{7, 23, 39, --- \} = \{ n_4 : n_4 = (2^4 k_4 + 7), k \geq 0. \}$

$1n, n_{41}, k_4$ can be odd or even

Therefore let, $n_5 = 2^5 k_5 + 15$ & $n_{51} = 2^5 k_5 + 31$

$n_5 = 2^5 k_5 + 2^4 - 1$

$f(n_5, 6) = 4(3^3 k_5 + 3^2 + 3 + 1) + 1$

Therefore, $A(n_5) = B(n_5)$.

Thus, the following set is generated.

$N(2^5) = \{15, 47, 79, --\} = \{ n_5 : n_5 = (2^5 k + 15), k \geq 0. \}$

Let, $n_r = 2^r k_r + 2^{r-1} - 1$ & $n_{r1} = 2^r k_r + 2^r - 1$.

$(r - 2) : -n_r = 2^r k_r + 2^{r-1-1}$

$f(n_r, 2r - 4) = 4(3^{r-2} k_r + 3^{r-3} + 3^{r-4} + -- + 1) + 1$

Therefore, $A(n_r) = B(n_r)$.

Thus, the following set is generated.

$N(2^r) = \{ (2^{r-1} - 1), --, (2^{r-1} - 1 + k2^r), -- \}$.

$N(2^r) = \{ n_r : n_r = (2^r k_r + 2^{r-1} - 1), k \geq 0. \}$

## CONCLUSION

All these sets put together constitutes the master set N (2)

$N(2) \equiv \{ N(2^2) \cup N(2^3) \cup ---- \cup N(2^r) --- : r \to \infty \}$.

The Set, $N(2) = \{1, 3, 5, ---\} = \{ n_2 : n_2 = (2k + 1), k \geq 0\}$

And $A(n_2) = B(n_2)$.

The above set are represents all odd numbers. All even numbers tends to an odd number.

Therefore, $A(n) = B(n)$ & $n \in N$

Thus, Collatz's Conjecture, is proved for all positive integers.