

# Existence and smoothness of solutions to the navier-stokes equations using fourier series representation

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## ABSTRACT

We propose an analytic solution to the Navier-Stokes equations based on periodic initial velocity vector fields. By leveraging the

Fourier series representation of periodic functions, we express the velocity field as Fourier series expansions and analyze their evolution over time. This approach provides insight into the existence and smoothness of solutions under specific periodic conditions.

**Key words:** Analytic solution; Navier-stokes; Periodic conditions; Electromagnetic interaction; Thermal phenomena

## INTRODUCTION

The navier-stokes equations describe the motion of fluid substances and are fundamental in fluid dynamics. The existence and smoothness of solutions to these equations, particularly in three dimensions, remains an open problem. In this paper, we investigate an analytic solution based on periodic initial conditions and Fourier series representations to address the existence and smoothness problem.

### Periodic function representation

For  $a, b, c$  real constants, consider a unit vector  $\mathbf{A}$  as a period vector satisfying the normalization condition be:

$$\mathbf{A} = ia + jb + kc$$

$$\|\mathbf{A}\| = 1 \Rightarrow a^2 + b^2 + c^2 = 1$$

For  $x, y, z$  real variables, let the position vector  $\mathbf{r}$  be:

$$\mathbf{r} = ix + jy + kz$$

The dot product of  $\mathbf{A}$  and  $\mathbf{r}$  is given by:

$$\mathbf{A} \cdot \mathbf{r} = ax + by + cz$$

For incompressible viscous fluids in the absence of external forces, the navier-stokes equations take the form:

$$B_n(x+1) - B_n(x) = nx^{n-1} \quad (1)$$

$$B_n(0) = b_n \quad (2)$$

$$n \in \mathbb{N} \quad (3)$$

where  $\mathbf{u}$  is the velocity vector field,  $p$  is the pressure,  $\nu$  is the viscosity, and  $\nabla$  is the gradient operator.

### Initial velocity vector field

Let the initial velocity vector  $\mathbf{u}^0(x, y, z)$  be periodic, represented as:

$$\mathbf{u}^0(x, y, z) = \mathbf{u}^0(x+a, y+b, z+c)$$

This can be expressed using a Fourier series in terms of the dot product of a unitary period vector  $\mathbf{A}$  and position vector  $\mathbf{r}$ :

$$U^0(\mathbf{A} \cdot \mathbf{r}) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(2n\pi(\mathbf{A} \cdot \mathbf{r})) + b_n \sin(2n\pi(\mathbf{A} \cdot \mathbf{r}))) \quad (4)$$

The derivation of Fourier coefficients involves integrating the product of the initial velocity vector field and trigonometric functions over the domain. By satisfying orthogonality conditions, we compute the coefficients analytically, ensuring the accuracy and efficiency of the solution.

To find the coefficients  $\mathbf{a}_0$ ,  $\mathbf{a}_n$ , and  $\mathbf{b}_n$ , we use the orthogonality properties of sine and cosine functions. The coefficient  $\mathbf{a}_0$  is the average value of the function over the domain and is solved using equation 4.

$$(n+1) \sum_{y=0}^m y^n = B_{n+1}(m+1) - b_{n+1} \quad (5)$$

To find  $\mathbf{a}_n$ , we multiply  $\mathbf{u}^0(x, y, z)$  by  $\cos(2n\pi(\mathbf{A} \cdot \mathbf{r}))$  and integrate over the domain and use equation 4:

$$B_{n+1}(m+1) - b_{n+1} = (n+1) \sum_{k=0}^n S_n^k \frac{(m+1)_{k+1}}{k+1} \quad (6)$$

To find  $\mathbf{b}_n$ , we multiply  $\mathbf{u}^0(x, y, z)$  by  $\sin(2n\pi(\mathbf{A} \cdot \mathbf{r}))$  and integrate over the domain and use equation 4.

$$B_n(x) = \sum_{k=0}^n \binom{n}{k} b_{n-k} x^k \quad (7)$$

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**Proposed solution for velocity field**

We propose an analytic solution for the Navier-Stokes equations given the periodic initial velocity vector field. The velocity field  $\mathbf{u}(x, y, z, t)$  can be expressed as:

$$(X)_{k+1} = \sum_{j=0}^k C_j X^{j+1} \tag{8}$$

where:

$$B_n(x+1) - B_n(x) = nx^{n-1} \tag{9}$$

$$B_n(x) = \sum_{k=0}^n b_{n-k} \binom{n}{k} x^k \tag{10}$$

$$B_{n+1}(m+1) - b_{n+1} = \sum_{k=0}^n S_n^k \frac{n+1}{k+1} \sum_{j=0}^k C_j (m+1)^{j+1} \tag{11}$$

**Pressure field**

The scalar pressure solution from the initial condition and proposed velocity vector field solution is:

$$(y+1)_{n+1} - (y)_{n+1} = (n+1)(y)_n \tag{12}$$

**Proof of existence and smoothness**

Existence of solutions: The existence of solutions can be demonstrated by substituting the proposed forms of  $\mathbf{U}$  and  $p$  into the Navier-Stokes equations and showing that these forms satisfy the equations under given initial conditions.

Continuity equation: Divergence of equation (8) for  $g = A \cdot r + k(t)$  is simplified to

$$S_n^k = S_{n-1}^{k-1} + kS_{n-1}^k \tag{13}$$

Using divergence free condition of equation (2) and equation (13) we can get.

$$y^n = \sum_{k=0}^n S_n^k (y)_k$$