

Premises and simple estimation on proof of the positive mass theorem, and negative matter as unified dark matter and dark energy

Yi-Fang Chang

Chang YF. Premises and simple estimation on proof of the positive mass theorem, and negative matter as unified dark matter and dark energy. *J Pure Appl Math.* 2022; 6(6):15-21.

ABSTRACT

First, we found that some proofs of the positive mass theorem have all certain premises: an isolated gravitational system and infinite space, but both are all impossible. Therefore, the negative matter cannot be restricted. Second, we discuss some simple estimations of the positive matter theorem, which agree from astronomy to particles. Third, based on Dirac negative energy, Einstein mass-energy relation and principle of equivalence, we propose the negative matter as the simplest model of unified dark matter and dark energy. All theories are known, only mass includes positive and negative. Because there is repulsion between positive matter and negative matter, so which is invisible dark matter, and repulsion as dark energy. It may explain many phenomena of dark

matter and dark energy. Bondi's results are wrong. Fourth, we derive that the rotational velocity of galaxy is approximate constant. Assume that dark matter is completely the negative matter, so we may calculate an evolutionary ratio between total matter and usual matter from 1 to present 11.82 or 7.88. Fifth, we propose the mechanism of inflation as origin of positive-negative matters created from nothing, whose expansion is exponential due to strong interactions at small microscopic scales. Moreover, we research some theories of the negative matter, and obtain the quantitative relation of the negative mass and the cosmological constant, and predict a judgment test and other possible tests. The negative matter as a candidate of dark matter and dark energy is not only the simplest, and is calculable and testable, and may be changed and developed.

Key Words: *Positive mass theorem; Negative matter; Repulsion; Dark matter; Dark energy; Calculation; Inflation; Test.*

INTRODUCTION

Premises on proof of the positive mass theorem

The positive mass (energy) theorem is an important conjecture in mathematical physics, from which the negative matter seems to be not existence. But, we study carefully some proofs of the positive mass (energy) theorem, and found that these proof processes all have certain premises [1-7].

In 1979 Schoen and Yau proved first the positive mass conjecture in general relativity [1]. Let M be a space-time whose local mass density is non-negative everywhere. Then they prove that the total mass of M as viewed from spatial infinity must be positive unless M is the flat Minkowski space-time. In general relativity they provided that an isolated gravitating system having non-negative local mass density must have non-negative total mass, measured gravitationally at spatial infinity. But, two premises are all physically impossible: gravitation forms an isolated system, and space is infinity. In fact, this is a proof

that an isolated system of general relativity with a positive density must have a total positive mass-energy, or a proof that an isolated gravitational system guarantees that the positive density becomes a total positive mass-energy. They supposed

$$g_{ij} = \left(1 + \frac{M}{2r}\right)^4 \delta_{ij} + h_{ij} \quad (1)$$

holds and N_k has zero total mass. The positive mean curvature is related to the metric $d\bar{s}^2 = \phi^4 ds^2$, which should be ≥ 0 .

The positive-mass conjecture states that for a nontrivial isolated physical system, the total energy, which includes contributions from both matter and gravitation, is positive. This assertion has been demonstrated in the important case when the space-time admits a maximal slice [2,3].

Further, they extended this proof to the positive-action conjecture of

Department of Physics, Yunnan University, Kunming, 650091, China

Correspondence: Yi-Fang Chang, Department of Physics, Yunnan University, Kunming, 650091, China, e-mail yifangch@sina.com

Received: Nov 21, 2022, Manuscript No. puljpm-22-5684, Editor Assigned: Nov 23, 2022, PreQC No. puljpm-22-5684 (PQ), Reviewed: Nov 24, 2022, QC No. puljpm-22-5684 (Q), Revised: Nov 25, 2022, Manuscript No puljpm-22-5684 (R), Published: Nov 30, 2022, DOI-10.37532/2752-8081.22.6(6).15-21



This open-access article is distributed under the terms of the Creative Commons Attribution Non-Commercial License (CC BY-NC) (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits reuse, distribution and reproduction of the article, provided that the original work is properly cited and the reuse is restricted to noncommercial purposes. For commercial reuse, contact reprints@pulsus.com

Hawking for asymptotically Euclidean metric, and the path integral convergent in the Euclidean quantum gravity theory [4]. For an isolated physical system, an initial data set for a space-time consists of a three-dimensional manifold N , a positive definite metric g_{ij} , a

symmetric tensor h_{ij} , a local mass density

$$\mu = \frac{1}{2} [R - \sum_{i,j} h^{ij} h_{ij} + (\sum_i h_i^i)^2], \quad (2)$$

and a local current density J . When $M=0$, the space-time is the Minkowski space-time. It is usual result of general relativity [5].

Witten proposed a new proof of the positive energy theorem of classical general relativity. If the positive energy theorem was false and a state of negative energy existed in general relativity, Minkowski space would presumably be unstable and would decay into the negative energy state. The main importance of the positive energy theorem is that it is related to the stability of Minkowski space as the ground state of general relativity [6]. Parker and Taubes discussed Witten's proof of the positive energy theorem [7]. But, according to the gravitational force:

$$F = -\frac{G}{r^2} M_1 M_2, \quad (3)$$

there is still gravitational force between negative-negative matters, it is universal repulsive force between the positive and negative matters. Therefore, the positive and negative matters are two regions of topological separation in general case by different interactions Figure.1 [8-10].

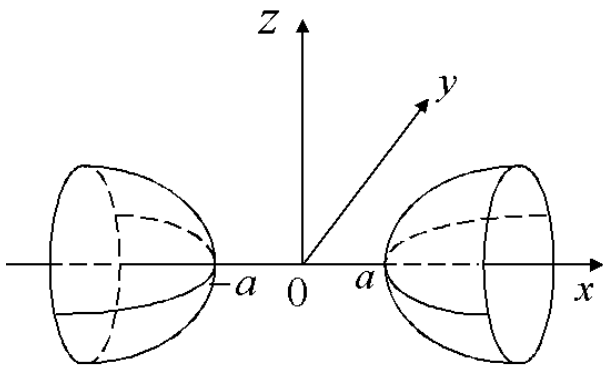


Figure 1) Positive and negative matters as two topological separation regions

Moreover, Townsend considered, for spacetimes of arbitrary dimension, the general two-derivative gravity plus scalar field theory allowing anti-de Sitter space as a solution, and studied the restrictions on the scalar field potential required by positivity of the energy. In particular, he rederived the perturbative stability criterion, and applied to higher dimensional supergravity theory [11]. Recently, Wang and Zhang established the positive energy theorem for weak asymptotically anti-de Sitter spacetime with distributional curvature under the weak dominant energy condition [12].

In a word, the positive energy theorem states that the total energy of a gravitating system, including the energy of the matter and also the

energy of the field, is always positive, if the matter contribution is positive. But, it is quite like a circular argument, which is admitted in Yau's book [13]. Conversely, if the matter contribution is negative, this proof will not hold.

Some simple estimations of the positive mass theorem

In this paper, we propose some simple estimation of the positive matter theorem.

In the modern cosmology, for a radiation-dominated universe of the big-bang cosmology, the total energy of usual matter is mainly the positive energy of photon Mc^2 . When the evolutionary process from inflation and radiation-dominated universe to the matter-dominated universe, the known total energy of usual baryon matter of non-relativity is:

$$Mc^2 - \frac{GmM}{r} \quad (4)$$

The positive mass theorem is simplified to $Mc^2 > GMm/r$,

i.e., $c^2 > Gm/r$. This constant $c^2 / G = 1.35 \times 10^{27} \text{ kgm}^{-1}$ is very big [14,15].

In astronomy, Solar mass: Solar radius is

$$M / R = 2.848 \times 10^{21} \text{ kgm}^{-1}$$

. For the solar system R is bigger, M/R is smaller. For black hole, $r = 2Gm/c^2$ so $1 > 0.5$ hold always. For Galactic System

$$M / R \approx 4 \times 10^{42} \text{ kg} / 2.85 \times 10^{21} \text{ m} = 1.4 \times 10^{21} \text{ kgm}^{-1}$$

For the particles and nuclei,

$$M / R = n(1.67 \times 10^{-27} \text{ kg}) / 10^{-15} \text{ m} = 1.67n \times 10^{-12} \text{ kgm}^{-1}$$

These all comply with the positive mass theorem.

But, the known total mass of Universe is $M = 2 \times 10^{53} \text{ kg}$ and

corresponding scale is $R = 4.2 \text{ Gpc} = 1.3 \times 10^{26} \text{ m}$, so

$$M / R = 1.538 \times 10^{27} \text{ kgm}^{-1}$$

. Both are comparable, and M/R is slightly big. It corresponds just to the acceleration of the universe and dark energy [16-17].

Dark matter and dark energy

Now dark matter and dark energy are always basic focus in astronomy, and total physics.

They are also the greatest mysteries in physics. General hypothesis is that dark matter and dark energy are two different concepts.

Since 1970 to 1978, Rubin, et al., confirmed the existence of dark matter for ten spiral galaxies [18]. Further, the dark matter in the Galaxy, in group of galaxies and cluster of galaxies, in the universe, is confirmed by the mass-to-light ratio and the galactic rotational curves, etc [19].

The dark matter may become huge conglomeration, and is possibly

the Weakly Interacting Massive Particle (WIMP), neutrino with mass, baryonic dark matter and nonbaryonic dark matter, monopole, supersymmetric dark matter, axion, etc [20-23].

The simplest kind of dark matter model is to add phenomenally a real scalar field as the dark matter field in the standard model [24,25]. Clowe, et al., proved a direct empirical proof of the existence of dark matter [26]. Gianfranco, et al., discussed history of dark matter [27].

Recently, in Nature there are two reports: "Tighter limits on dark matter" and "Dark-matter evidence weakens". Both are based on search for ultralight scalar dark matter with atomic spectroscopy, and 7.1 keV sterile neutrino constraints from X-ray observations of 33 clusters of galaxies with Chandra ACIS [28-29]. Vermeulen, et al., discussed direct limits for scalar field dark matter from a gravitational-wave detector [30]. Argüelles, et al., studied dark matter annihilation to neutrinos [31]. Allali, et al. discussed general relativistic decoherence with applications to dark matter detection [32]. Arcadi, et al., searched muon $g-2$ and B anomalies from dark matter [33]. Bringmann, et al., discussed dark matter from exponential growth [34]. Holst, et al., proposed simplest and most predictive model of muon $g-2$ and thermal dark matter [35].

The dark energy as a huge repulsive force is proposed in order to explain the acceleration of inflation in the universe [17], and may unify many different results of observations. The dark energy seems to be the energy of vacuum, and be zero mass. Usually assume that the dark energy connects with the cosmological constant, and is a dynamical scalar field (quintessence model). Scherrer proposed a new k-essence models unified dark matter and dark energy [36]. Caldwell, et al., discussed observations continue to indicate that the Universe is dominated by invisible dark matter and dark energy [37].

Recently Foster, et al., searched decaying dark matter with XMM-Newton Blank-Sky Observations [38]. PandaX Collaboration, et al., searched some experiments on dark matter [39-42]. But, so far many models on dark matter and dark energy are not testability.

For new data, usual visible matter is 4.84%, dark matter is 25.96%, and dark energy is 69.2% [43-44].

Some believe that dark energy distributes uniformly in the whole space, and its interactions are repulsive. But, in the solar system dark energy and dark matter cannot exist, since in which general relativity is very exact. The negative matter should be groups, so complete uniformity is impossible. Recently, astronomers find more galaxies in the Universe without any dark matter. Guo, et al. reported 19 dwarf galaxies that could consist mainly of baryons, and provided observational evidence that could challenge the formation theory of low-mass galaxies within the framework of standard cosmology [45].

3 basic principles of negative matter as unified dark matter-energy

In 1928 Dirac predicted anti-particles and the negative energy state from his equation, and he emphasized: "we cannot ignore the negative energy states" [46]. In order to prevent to jump continuously from positive energy state to negative energy state in the quantum

theories, and keep the stability of world, Dirac proposed that as long as suppose that all the states of negative energy are occupied except perhaps a few of small velocity. The vacuum of the realistic world has already been filling with all negative energy state, such the Pauli exclusion principle will come into play and prevent more than one electron going into any one state, and avoid this jumping difficulty. It is namely the well-known Dirac negative energy sea and whose vacancy or hole is an anti-particle (or opposite particle). From this the annihilation and creation between positive and opposite particles may be predicted. There is exact description in <The Principles of Quantum Mechanics> [47]. But, it prevent only jump of fermions, but cannot be applied to bosons. Therefore, the stability problem exists still. In fact, the negative energy state appears in all relativity theories as $E = \pm\sqrt{p^2c^2 + m^2c^4}$, even also in the classical theory.

From 2007 we proposed that the negative matter developed the Dirac negative energy may unify dark matter and dark energy, in which the anti-(opposite) matter and the negative matter are different. The anti-matter is that some properties of matter are opposite, for instance, charge, baryon number, lepton number, strangeness number and so on, but their masses and total energy are still positive. These particles include positron and various anti-particles [48-52,8-10]. The existence of these particles is already verified. Both positive and opposite matters meet to annihilate to photons with conservation of energy and zero-charge. The negative matter has a negative mass and total energy. The creation of negative matter is difficult, but its existence will be stable.

Based on Dirac negative energy state, we proposed the negative matter, whose key is negative mass. Based on $M \rightarrow M_+ - M_-$ and Eq.(3), all theories are all known. The positive and negative matters are two regions of topological separation in general case by different interactions, so the negative matter is invisible dark matter. The negative matter should be a necessary development of Dirac theory (Figure 1).

According to the principle of equivalence in general relativity, inertial mass and gravitational mass must be equal always. Based on Eq.(3), there are only three cases: positive and positive matters, positive and negative matters, negative and negative matters. But, for negative mass Bondi proposed three kinds of mass: inertial, passive gravitational, and active gravitational mass, and there are four cases. Such Bondi believes that the positive body will attract the negative one (since all bodies are attracted by it!), etc. It is a fallacy with contradictions [53].

According to Einstein mass-energy relation $E = mc^2$ dark matter and dark energy should be unified, and it agrees with Occam's Razor [10]. Repulsion between positive matter and negative matter shows dark energy. This is the simplest candidate of dark matter, and can be unified dark energy, and may explain many phenomena of dark matter and dark energy.

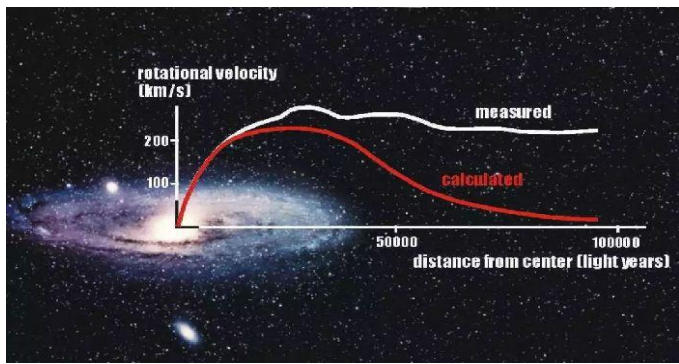


Figure 2) In Andromeda Galaxy the curve of the rotational velocity with distance. The red curve is the theoretical calculated prediction of the no-dark matter, and the white curve is the measured curve.

In Figure 2 the difference between the two curves is one of the key evidence for the existence of dark matter. According to classical mechanics the rotational velocity is:

$$\frac{1}{R}V^2 = \frac{GM}{R^2} \quad (5)$$

The measured curve for large scale $R > R_0$ is:

$$V^2 = \frac{GM}{R} \approx \text{constant} \quad (6)$$

If the negative matter is introduced $M \rightarrow M_+ - M_-$, the

equation (5) will become: $\frac{G}{R^2}(M_+ - M_-) = \frac{1}{R}V^2$ (7) The

total mass of the spherality galaxy inside radius R is:

$$M(R) = M_+ - M_- = \int_0^R (\rho_+ - \rho_-) dV = (\rho_+ - \rho_-) \int_0^R 4\pi r^2 dr = \frac{4\pi}{3} R^3 (\rho_+ - \rho_-) \quad (8)$$

$$\frac{dV}{dt} = -\frac{G}{R^2}(M_+ - M_-) = -\frac{4\pi}{3} GR(\rho_+ - \rho_-) \quad (9)$$

For the plane disk-like galaxy

$$M(R) = (\rho_+ - \rho_-) \int_0^R 2\pi r dr = \pi R^2 (\rho_+ - \rho_-) \quad (10)$$

$$\frac{dV}{dt} = -\pi G(\rho_+ - \rho_-) \quad (11)$$

If $\rho_+ \approx \rho_-$, and $M_+ \approx M_-$, $dV/dt \approx 0$, integral derive V and

V^2 are approximate constants.

Basic Calculations of Negative Matter as Unified Dark Matter-Energy

For a radiation-dominated universe the total energy of positive and negative matters should be [14,15].

$$M_+c^2 - M_-c^2 \quad (12)$$

Because inflation is origin of nothing, the total energy should be zero, i.e., $M_+ = M_-$

When the evolutionary process from inflation and radiation-dominated universe to the matter-dominated universe, the known total energy of usual baryon matter of non-relativity is:

$$M_+c^2 - \frac{GM_+^2}{R_+} \quad (13)$$

Assume that dark matter and dark energy are completely the negative matter, so the total energy includes three parts: one of the positive matter, one of negative matter, and their repulsion force:

$$E_t = M_+c^2 - \frac{GM_+^2}{R_+} + (-M_-c^2 - \frac{GM_-^2}{R_-}) + \frac{GM_+M_-}{R_{\pm}} \quad (14)$$

Both ratio is:

$$\frac{M_+c^2 - \frac{GM_+^2}{R_+} + (-M_-c^2 - \frac{GM_-^2}{R_-}) + \frac{GM_+M_-}{R_{\pm}}}{M_+c^2 - \frac{GM_+^2}{R_+}} \quad (15)$$

We suppose that for early inflation cosmology the positive matter and the negative matter have the same mass $M_+ = M_- = M$, and both are separated. In order to simplify assume that positive and negative matters form two identical spheres, respectively, so

$R_+ = R_- = R, R_{\pm} = 2R$. Such Eq. (15) is simplified to:

$$\frac{-\frac{3GM^2}{2R}}{Mc^2 - \frac{GM^2}{R}} = \frac{3}{2} \frac{GM}{GM - Rc^2} \quad (16)$$

Based on the known total mass of Universe and corresponding scale, so $Rc^2 = 1.17 \times 10^{43} m^3 s^{-2}$, and

$$GM = 1.34 \times 10^{43} m^3 s^{-2} \quad (17)$$

A simple calculation obtains $\frac{3}{2} \frac{GM}{GM - Rc^2} = 11.82$. Of

course, the actual situations are more complicated. But, this is a model that can be computed and compared, and may also be developed [16].

If $R_{\pm} = R$, positive matter and negative matter are mix together in the same volume, so Eq.(15) is simplified to:

$$\frac{-\frac{GM^2}{R}}{Mc^2 - \frac{GM^2}{R}} = \frac{GM}{GM - Rc^2} = 7.88 \quad (18)$$

According to new data, ratio between usual matter, total matter and dark energy is 4.84:30.8:69.2, so $30.8/4.84=6.36$, and $69.2/4.84=14.3$. Since G, M, c are invariance, R is bigger, this ratio is bigger [43,44].

When $\rho_+ = \rho_-$, $M_+ = M_-$, the expansion rate is unchanged.

$\rho_+ - \rho_- > 0, M_+ - M_- > 0, dV/dt < 0$, the expansion rate is deceleration.

$\rho_+ - \rho_- < 0, M_+ - M_- < 0, dV/dt > 0$, the expansion rate is acceleration.

Other deductions and possible tests on negative matter

It is known that the gravitational field equations with the cosmological constant are:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi k T_{\mu\nu}. \quad (19)$$

We proposed the field equations of general relativity on the negative matter [48-50],

$$G_{\mu\nu} = 8\pi k (T_{\mu\nu} - T'_{\mu\nu}). \quad (20)$$

So the cosmological constant Λ corresponds to the negative matter, i.e., $\Lambda = 8\pi k T'_{\mu\nu} / g_{\mu\nu}$.

Here $\Lambda g_{\mu\nu}$ corresponds to the negative energy state and vacuum energy (Dirac sea), and is consistent with conformal gravity theory.

Caldwell proposed phantom as cosmological consequences of a dark energy component with super-negative equation of state, whose cosmic energy density has negative pressure. The total energy is negative, so it is namely a type of negative matter. Then phantom becomes an important dark energy model [54].

In cosmology it is an important progress that Guth proposed inflation whose time origin is from 10^{-32} s, and cosmic scale factor exponential expansion $a(t) \approx e^{Ht}$ [55]. Then Linde and Albrecht, et al. [56-57], proposed the chaotic inflation. We proposed the mechanism of inflation as origin of positive-negative matters created from nothing at the same time, whose quantum fluctuations correspond just to the chaotic inflation [48,8-10]. It is a Planck time 10^{-43} s, and length 10^{-33} cm. At this very small space the positive matter and negative matter are the very strong repulsive interaction, and the exponential inflation is just a form of the strong interaction:

$$F = -g^2 \frac{e^{-kr}}{r^2}. \quad (21)$$

Here the positive matter is g , and the negative matter is $-g$, so $F > 0$ is a huge strong repulsive force for the length inside 10^{-13} cm. When the scale is bigger than one of the strong interaction, the inflation finishes, and the positive matter and opposite matter will form two regions of topological separation repulsed each other Figure 1.

Based on observations of a remarkable cosmic structure called the bullet cluster, Bradac, et al., discovered that this structure is actually two clusters of galaxies passing through one another [57-58]. The new research is the first to detect luminous matter and dark matter independent of one another, with the luminous matter clumped together in one region and the dark matter clumped together in another. These observations demonstrate that there are two types of matter: one visible and one invisible. In 2007, COSMOS obtained

first three-dimensional distribution map of dark matter on world [59-60].

We researched the most complete theory of the negative matter, and its quantum theory, and corresponding Lobachevskian geometry. We proposed a judgment test for the negative matter as dark matter, and other possible tests [9,8-10].

Various positive matter and black hole exhibit the gravitational lensing effect. The negative matter will be the repulsive lensing, and will form the bigger Einstein ring. Both should be different in observations (Figure 3).

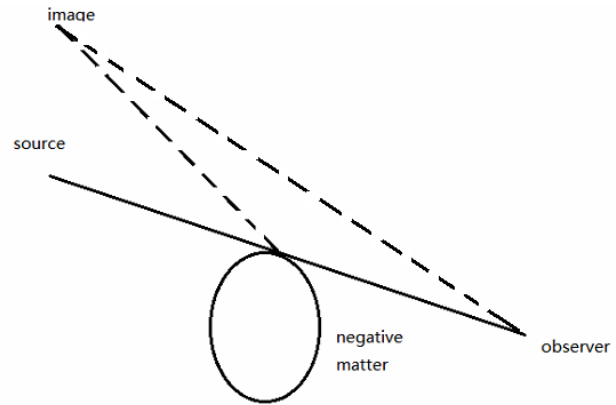


Figure 3) Repulsive lensing

Recently, astronomers observed a super-huge hole about 2 billion light-years scale (Figure 4). This is most likely an invisible region of the negative matter, because it is not the spherical symmetry, and is not found intense activity if as a huge black hole.



Figure 4) A super-huge hole in Universe

CONCLUSION

It is a notable process that the positive energy from a conjecture became to a theorem, and then to a restriction for the negative matter. But, the premises on proof of the positive energy conjecture cannot rule out Dirac's genius prophecy on the negative energy state.

Further, a completely similar approach can prove that the negative matter region is always the total negative energy, and gravity is also negative energy, so the total negative energy is greater.

Chang

Usually, the positive matter and the negative matter are two regions separated each other, and both are all stable. When they annihilate, both are certainly unstable.

Generally, the whole matter space is divided into:

Positive matter region, the gravitational energy is negative, and the total energy is positive.

The negative matter region, the gravitational energy is negative, and the total energy is greater negative energy.

A region between positive and negative matters, the repulsion energy is positive.

The total region of the three is mixed, the positive matter and negative matter are equal, the double gravity is greater than the repulsion, the total energy should be negative, so that the whole universe accelerates the expansion, and corresponds to the dark energy as a huge repulsive force.

In fact, some physicists researched already the negative energy and various related problems.

Early in 1898 Arthur Schuster conjectured the existence of new sun with the negative mass. The positive, opposite, and negative, negative-opposite matters may form just the most perfect symmetrical world.

In a word, the negative matter as a candidate of unified dark matter and dark energy is not only the simplest and is consistent with Occam's Razor, and is calculable and testable, and may be changed and developed.

REFERENCES

1. Schoen P, Yau S T. On the proof of the positive mass conjecture in general relativity. *Comm Math Phys.* 1979;65:45-76.
2. Schoen P, Yau ST. Positivity of the total mass of a general space-time. *Phys Rev Lett.* 1979;43(20):1457-59
3. Schoen P, Yau S T. Proof of the positive mass theorem. II *Comm Math Phys.* 1981;79:231-60
4. Schoen P, Yau S T. Proof of the positive action conjecture in quantum relativity. *Phys Rev Lett.* 1979;42:547-48
5. Schoen R, Yau S T. The energy and the linear momentum of space-times in general relativity. *Comm Math Phys.*1981;79 (1): 47-51 [Google Scholar] [Crossref]
6. Witten E. A new proof of the positive energy theorem. *Comm Math Phys.* 1981;80(3):381-402
7. Parker T, Taubes C. On Witten's proof of the positive energy theorem. *Comm Math Phys.* 1982;84(2):223-38
8. Chang Y F. Negative matter as unified dark matter and dark energy, and possible tests. *Had J.* 2017;40(3):291-308
9. Chang Y F. Negative matter as unified dark matter and dark energy: simplest model, theory and nine tests. *Int J Fundam Phys Sci.* 2020;10(4):40-54
10. Chang Y F. Development of matter and testable negative matter as unified dark matter and dark energy. *Philos Study.* 2021;11(7):517-26
11. Townsend P K. Positive energy and the scalar potential in higher dimensional (super) gravity theories. *Phys Lett B.* 1984;148(1-3):55-59
12. Wang Y, Zhang X. Positive energy theorem for asymptotically anti-de Sitter spacetimes with distributional curvature. *Int J Math.* 2019;30(13):1940003 [Google Scholar] [Crossref]
13. Yau S T, Nadis S. *The Shape of Inner Space.* Brockman Inc. 2010.
14. Dodelson S. *Modern Cosmology.* Acad Press. 2003
15. Weinberg S. *Cosmology.* Oxf Univ Press. 2008
16. Perkins D. *Particle Astrophysics (Second Edition).* Oxf Univ Press. 2003
17. Peebles P J E, Bharat R. The cosmological constant and dark energy. *Rev Mod Phys.* 2003;75(2):559-606
18. Rubin V C, Ford W K, Thonnard N. Extended rotation curves of high-luminosity spiral galaxies. IV. Systematic dynamical properties. *Astrophys J.* 1978;225:107-11
19. Binney J, Tremaine S. *Galactic Dynamics.* Princet Univ Press. 1987
20. Copi C J, Schramm D N, Turner M S. Big-bang nucleosynthesis and the baryon density of the universe. *Sci.* 1995;267(5195):192-99
21. Jungman G, Kamionkowski M, Griest K. Supersymmetric dark matter. *Phys Reports.* 1996;267(5):195-373
22. Shellard E P S, Battye R A. Origin of dark matter axions. *Phys Reports.* 1998;307(4):227-34
23. Salemi C P, Foster J W, Ouellet J L, et al. Search for low-mass axion dark matter with ABRACADABRA-10 cm. *Phys Rev Lett.* 2021;127(8): 081801
24. McDonald J. Gauge singlet scalars as cold dark matter. *Phys Rev.* 1994;D50(6):3637-49
25. Barger V, Langacker P, McCaskey M, et al. CERN LHC phenomenology of an extended standard model with a real scalar singlet. *Phys Rev.* 2008;77(3):035005.
26. Clowe D. A direct empirical proof of the existence of dark matter. *Ap J.* 2006;648(2):L109-L113
27. Gianfranco B, Dan H. History of dark matter. *Rev Mod Phys.* 2018;90(4):045002.
28. Van T K, Leefer N, Bougas L, et al. Search for ultralight scalar dark matter with atomic spectroscopy. *Phys Rev Lett.* 2015;115(1):011802.

29. Hofmann F, Sanders J S, Nandra K, et al. 7.1 keV sterile neutrino constraints from X-ray observations of 33 clusters of galaxies with Chandra ACIS. *A&A*. 2016;592:A112
30. Vermeulen S M, Relton P, Grote H, et al. Direct limits for scalar field dark matter from a gravitational-wave detector. *Nat*. 2021;600(7889):424-28.
31. Argüelles C A, Diaz A, Kheirandish A, et al. Dark matter annihilation to neutrinos. *Rev Mod Phys*. 2021;93(3):035007
32. Allali I J, Hertzberg M P. General relativistic decoherence with applications to dark matter detection. *Phys Rev Lett*. 2021;127(3):031301
33. Arcadi G, Calibbi L, Fedele M, et al. Muon $g-2$ and B anomalies from dark matter. *Phys Rev Lett*. 2021;127(6):061802
34. Bringmann T, Depta P F, Hufnagel M, et al. Dark matter from exponential growth. *Phys Rev Lett*. 2021;127(19):191802 [Google Scholar] [Crossref]
35. Holst I, Hooper D, Krnjaic G. Simplest and most predictive model of muon $g-2$ and thermal dark matter. *Phys Rev Lett*. 2022;128(14): 141802.
36. Scherrer R J. Purely kinetic k essence as unified dark matter. *Phys Rev Lett*. 2004;93(1):011301
37. Caldwell R, Kamionkowski M. Dark matter and dark energy. *Nat*. 2009;458(7238):587-89.
38. Foster J W, Kongsore M, Dessert C, et al. Deep search for decaying dark matter with XMM-Newton Blank-Sky Observations. *Phys Rev Lett*. 2021;127(5):051101.
39. Meng Y. (PandaX-4T Collaboration), Dark matter search results from the PandaX-4T Commissioning Run. *Phys Rev Lett*. 2021;127(26):261802.
40. Adhikari P. (DEAP Collaboration), First direct detection constraints on Planck-scale mass dark matter with multiple-scatter signatures using the DEAP-3600 Detector. *Phys Rev Lett*. 2022;128(1):011801.
41. Aiello L, Richardson J W, Vermeulen SM, et al. Constraints on scalar field dark matter from Colocated Michelson Interferometers. *Phys Rev Lett*. 2022;128(12):121101 [Google Scholar] [Crossref]
42. Cui X. (PandaX-II Collaboration), Search for cosmic-ray boosted Sub-GeV dark matter at the PandaX-II Experiment. *Phys Rev Lett*. 2022;128(17):171801.
43. Adam R. (Planck Collaboration), Planck 2015 results-I. Overview of products and scientific results. *Astron & Astrophys*. 2016;594: A1-A13.
44. Tanabashi M. Particle data group. *Phys Rev*. 2018;D98:030001.
45. Guo Q, Hu H, Zheng Z, et al. Further evidence for a population of dark-matter-deficient dwarf galaxies. *Nat Astro*. 2020;4(3):246-51.
46. Dirac P A M. The theory of electrons and protons. *Proc Roy Soc*. 1930;A126:365-69.
47. Dirac P A M. *The Principles of Quantum Mechanics*. Oxford. (1958).
48. Chang Y F. Negative matter, repulsion force, dark matter and inflation cosmos, Higgs mechanism. *arXiv*. 2007;0705:2908.
49. Chang Y F. Negative matter, dark matter and theoretical test. *Int Revi Phy*. 2011;5(6):340-45.
50. Chang Y F. Field equations of repulsive force between positive-negative matter, inflation cosmos and many worlds. *Int. J. Mod. Theor. Phys*. 2013;2(2):100-17.
51. Chang Y F. Astronomy, black hole and cosmology on negative matter, and qualitative analysis theory. *Int J Mod Appl Phys*. 2014;4(2):69-82
52. Chang Y F. Negative matter as dark matter, and its judgment test and calculation of ratio. *Int J Mod Appl Phys*. 2019;9(1):1-12 [Google Scholar] [Crossref]
53. Bondi H. Negative mass in general relativity. *Rev Mod Phys*. 1957;29:423-28.
54. Caldwell R R. A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state. *Phys Lett*. 2002;B545:23-9 .
55. Guth A G. The inflationary universe: a possible solution to the horizon and flatness problems. *Phys Rev*. 1981;D23(2):347-56.
56. Linde A D. A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. *Phys Lett*. 1982;B108(6):389-93.
57. Albrecht A, Steinhardt P J. Cosmology for grand unified theories with radiatively induced symmetry breaking. *Phys Rev Lett*. 1982;48(17):1220-23.
58. Bradac M, Clowe D, Gonzalez A H, et al. Strong and weak lensing united. III. Measuring the mass distribution of the merging galaxy cluster 1ES 0657-558. *Ap J*. 2006;652(2):937-47.
59. Massey R, Rhodes J, Ellis R, et al. Dark matter maps reveal cosmic scaffolding. *Nat*. 2007;445(7125):286-90.
60. Schuster A. Potential matter—A holiday dream. *Nat*. 1898;58(1503):367.